Rates of Multipartite Entanglement Transformations

A. Streltsov,[1,2,3,4] C. Meignant,[5,4] and J. Eisert[4]
[1]Centre for Quantum Optical Technologies, Centre of New Technologies, University of Warsaw, Banacha 2c, Warsaw 02-097, Poland
[2]Faculty of Applied Physics and Mathematics, Gdańsk University of Technology, Gdańsk 80-233, Poland
[3]National Quantum Information Centre in Gdańsk, Sopot 81-824, Poland
[4]Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, Berlin 14195, Germany
[5]Laboratoire d'Informatique de Paris 6, CNRS, Sorbonne Université, 4 place Jussieu, Paris 75005, France

(Received 4 November 2017; revised 18 October 2019; accepted 28 July 2020; published 17 August 2020)

The theory of the asymptotic manipulation of pure bipartite quantum systems can be considered completely understood: the rates at which bipartite entangled states can be asymptotically transformed into each other are fully determined by a single number each, the respective entanglement entropy. In the multipartite setting, similar questions of the optimally achievable rates of transforming one pure state into another are notoriously open. This seems particularly unfortunate in the light of the revived interest in such questions due to the perspective of experimentally realizing multipartite quantum networks. In this Letter, we report substantial progress by deriving simple upper and lower bounds on the rates that can be achieved in asymptotic multipartite entanglement transformations. These bounds are based on ideas of entanglement combing and state merging. We identify cases where the bounds coincide and hence provide the exact rates. As an example, we bound rates at which resource states for the cryptographic scheme of quantum secret sharing can be distilled from arbitrary pure tripartite quantum states. This result provides further scope for quantum internet applications, supplying tools to study the implementation of multipartite protocols over quantum networks.

DOI: 10.1103/PhysRevLett.125.080502

Entanglement is the feature of quantum mechanics that renders it distinctly different from a classical theory [1]. It is at the heart of quantum information science and technology as a resource that is used to accomplish task (and is increasingly also seen as an important concept in condensed-matter physics). Given its significance in protocols of quantum information, it hardly surprises us that, already early in the development of the field, questions were asked how one form of entanglement could be transformed into another. It was one of the early main results of the field of quantum information theory to show that all pure bipartite states could be asymptotically reversibly transformed to maximally entangled states with local operations and classical communication (LOCC) at a rate that is determined by a single number [2]: the entanglement entropy, the von-Neumann entropy of each reduced state. This insight makes the resource character of bipartite entanglement most manifest: the entanglement content is given simply by its content of maximally entangled states, and each form can be transformed reversibly into another and back.

The situation in the multipartite setting is significantly more intricate, however [3–5]. The rates that can be achieved when aiming at asymptotically transforming one multipartite state into another with LOCC are far from clear. It is not even understood what the "ingredients" of multipartite entanglement theory are [4,6], so the basic units of multipartite entanglement from which any other pure state can be asymptotically reversibly prepared. This state of affairs is unfortunate, and even more so since multipartite states come again more into the focus of attention in the light of the observation that elements of the vision of a quantum network—or the "quantum internet" [7]—may become an experimental reality in the not too far future. It is not that multipartite entanglement ceases to have a resource character. For example, Greenberger-Horne-Zeilinger (GHZ) states are known to constitute a resource for quantum secret sharing [8,9], which is probably the best known multipartite cryptographic primitive. Progress on stochastic conversion for several copies of multipartite states was made recently [10,11]. However, given a collection of arbitrary pure states, it is not known at what rate such states could be asymptotically distilled under LOCC.

In this Letter, we report substantial progress on the old question of the rate at which GHZ and other multipartite states can be asymptotically distilled from arbitrary pure states. In this effort, much of the technical substance can be delegated to the powerful machinery of entanglement combing [12], putting it here into a fresh context, which in turn can be seen to derive from quantum state merging [13,14], assisted entanglement distillation [15,16], and time-sharing, meaning, using resource states in different roles in the asymptotic protocol. The basic insight underlying the analysis is that entanglement combing provides a reference, a helpful normal form rooted in the better understood theory of bipartite entanglement, that can be used in order to assess rates of asymptotic multipartite state conversion. Basically, putting entanglement combing to good work, therefore, we are in the position to make significant progress on the question of entanglement transformation rates in a general setting.

Multipartite state conversion.—We consider the problem of converting an n-partite state ρ into σ via n-partite LOCC. In particular, we are interested in the optimally achievable asymptotic rate for this procedure, which can be formally defined as

$$R(\rho\to\sigma)=\sup\left\{r:\lim_{k\to\infty}\left(\inf_{\Lambda}\left\|\Lambda(\rho^{\otimes k})-\sigma^{\otimes\lfloor rk\rfloor}\right\|_1\right)=0\right\}. \tag{1}$$

Here, Λ reflects an n-partite LOCC operation and $||M||_1 = \mathrm{Tr}\sqrt{M^\dagger M}$ denotes the trace norm. This problem has a known solution in the bipartite case $n=2$ for conversion between arbitrary pure states $\psi^{AB}\to\phi^{AB}$, rooted in Shannon theory. The corresponding rate in this case can be written as [2]

$$R(\psi^{AB}\to\phi^{AB})=\frac{S(\psi^A)}{S(\phi^A)}, \tag{2}$$

where $S(\rho)=-\mathrm{Tr}(\rho\log_2\rho)$ is the von Neumann entropy. Moreover, ψ^{AB} indicates that the state is shared between parties referred to as Alice and Bob, while ψ^A reflects the reduced state of Alice.

This simple picture ceases to hold in any setting beyond the bipartite one. Indeed, significantly less is known in the multipartite setting for $n\geq 3$ [3]. Needless to say, the bipartite solution (2) readily gives upper bounds on the rates in multipartite settings. For example, for conversion between tripartite pure states $\psi^{ABC}\to\phi^{ABC}$, it must be true that

$$R(\psi^{ABC}\to\phi^{ABC})\leq\min\left\{\frac{S(\psi^A)}{S(\phi^A)},\frac{S(\psi^B)}{S(\phi^B)},\frac{S(\psi^C)}{S(\phi^C)}\right\}. \tag{3}$$

This follows from the fact that any tripartite LOCC protocol is also bipartite with respect to any of the bipartitions. If the desired final state ϕ^{ABC} is the GHZ state with state vector $|\mathrm{GHZ}\rangle=(|0,0,0\rangle+|1,1,1\rangle)/\sqrt{2}$, the bound in Eq. (3) is known to be achievable whenever one of the reduced states ψ^{AB}, ψ^{BC}, or ψ^{AC} is separable [16].

We also note that for some states the bound in Eq. (3) is a strict inequality. This can be seen by considering the scenario where each of the parties holds two qubits, respectively. Consider now the transformation

$$|\mathrm{GHZ}\rangle^{A_1B_1C_1}\otimes|\mathrm{GHZ}\rangle^{A_2B_2C_2}\to |\Phi^+\rangle^{A_1B_1}\otimes|\Phi^+\rangle^{A_2C_1}\otimes|\Phi^+\rangle^{B_2C_2}, \tag{4}$$

i.e., the parties aim to transform two GHZ states into Bell state vectors $|\Phi^+\rangle=(|0,0\rangle+|1,1\rangle)/\sqrt{2}$ which are equally distributed among all the parties. It is straightforward to check that in this case the bound in Eq. (3) becomes $R\leq 1$. However, the bound is not achievable, as the aforementioned transformation cannot be performed with unit rate [17].

Lower bound on conversion rates for three parties.—The above discussion suggests that the bound in Eq. (3) is a very rough estimate for general transformations and is saturated only for very specific sets of states, having zero volume in the set of all pure states. We will see below that this is not the case: there exist large families of tripartite pure states which saturate the bound (3). This will follow from a very general lower bound on conversion rate, which will be presented below in Theorem 1. The methods developed here build upon and further develop the machinery of *entanglement combing*, which has been introduced and studied for general n-partite scenarios in Ref. [12]. In the specific tripartite setting, entanglement combing aims to transform the initial state ψ^{ABC} into a state of the form $\mu^{A_1B}\otimes\nu^{A_2C}$ with pure bipartite states μ and ν. The following Lemma restates the results from Ref. [12] in a form which will be suitable for the purpose of this Letter.

Lemma 1: conditions from tripartite entanglement combing.—The transformation

$$\psi^{ABC}\to\mu^{A_1B}\otimes\nu^{A_2C} \tag{5}$$

is possible via asymptotic LOCC if and only if

$$E(\mu^{A_1B})+E(\nu^{A_2C})\leq S(\psi^A), \tag{6a}$$

$$E(\mu^{A_1B})\leq S(\psi^B), \tag{6b}$$

$$E(\nu^{A_2C})\leq S(\psi^C), \tag{6c}$$

where $E(\phi^{XY}):=S(\phi^X)$ for pure states ϕ^{XY}. We refer to the Supplemental Material [18] for the proof of the Lemma. Using this result, we are now in position to present a tight lower bound on the transformation rate between tripartite pure states.

Theorem 1: lower bound for state transformations.—For tripartite pure states ψ^{ABC} and ϕ^{ABC}, the LOCC conversion rate is bounded from below as

$$R(\psi^{ABC}\to\phi^{ABC})\geq\min\left\{\frac{S(\psi^A)}{S(\phi^B)+S(\phi^C)},\frac{S(\psi^B)}{S(\phi^B)},\frac{S(\psi^C)}{S(\phi^C)}\right\}. \tag{7}$$

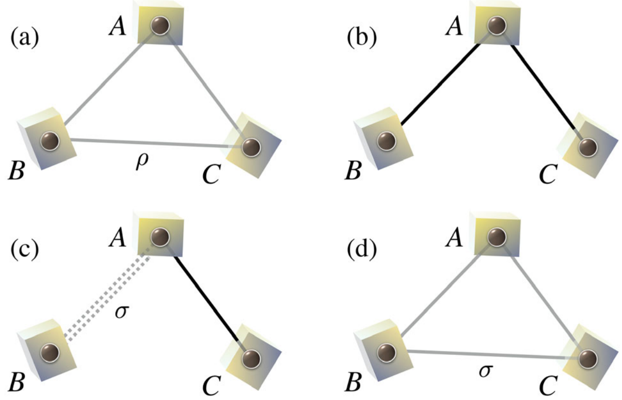

FIG. 1. Conversion of a multipartite resource state ρ (a) into the desired final state σ (d). The conversion is achieved via entanglement combing, i.e., via transforming the initial state ρ into singlets [black solid lines in (b)]. One of the singlets is then converted into the desired final state σ [gray dotted lines in (c)]. The remaining singlets [black solid line in (c)] are then used for teleporting the parts of σ to the remaining parties.

Proof.—We prove this bound by presenting an explicit protocol achieving the bound, which is also summarized in Fig. 1. In the first step, the parties apply entanglement combing $\psi^{ABC} \to \mu^{A_1B} \otimes \nu^{A_2C}$ in such a way that the following equalities are fulfilled for some $r \geq 0$,

$$E(\mu^{A_1B}) = rS(\phi^B), \qquad E(\nu^{A_2C}) = rS(\phi^C). \tag{8}$$

The significance of this specific choice will become clear in a moment. In the next step, Alice and Charlie apply LOCC for transforming the state ν^{A_2C} into the desired final state $\phi^{A_2A_3C}$. Since this is a bipartite LOCC protocol, the rate for this process is given by $E(\nu^{A_2C})/S(\phi^C)$. Note that, due to Eqs. (8), this rate is equal to r.

In a next step, Alice applies what is called Schumacher compression [19] to her register A_3. The overall compression rate per copy of the initial state ψ^{ABC} is given as

$$\tilde{r} = rS(\phi^{A_3}) = rS(\phi^B), \tag{9}$$

where in the last equality we used the fact that $S(\phi^{A_3}) = S(\phi^B)$. Due to Eqs. (8), this rate interestingly coincides with the entanglement of the state μ^{A_1B},

$$\tilde{r} = E(\mu^{A_1B}). \tag{10}$$

In a final step, Alice and Bob distill the states μ^{A_1B} into maximally entangled bipartite singlets, and use them to teleport [20,21] the (compressed) particle A_3 to Bob. Due to Eq. (10), Alice and Bob share exactly the right amount of entanglement for this procedure, i.e., the process is possible with rate one and no entanglement is left over. In summary, the overall protocol transforms the state ψ^{ABC} into ϕ^{ABC} at rate r.

To complete the proof, we will now show that r can be chosen such that

$$r = \min\left\{\frac{S(\psi^A)}{S(\phi^B) + S(\phi^C)}, \frac{S(\psi^B)}{S(\phi^B)}, \frac{S(\psi^C)}{S(\phi^C)}\right\}. \tag{11}$$

This can be seen directly by inserting Eqs. (8) into Eqs. (6). In particular, the rate r can attain any value which is simultaneously compatible with inequalities

$$r \leq \frac{S(\psi^A)}{S(\phi^B) + S(\phi^C)}, \quad r \leq \frac{S(\psi^B)}{S(\phi^B)}, \quad r \leq \frac{S(\psi^C)}{S(\phi^C)}. \tag{12}$$

This completes the proof of the theorem.

We stress some important aspects and implications of this theorem. Whenever the minimum in Eq. (7) is attained on the second or third entry, the lower bound coincides with the upper bound in Eq. (3). This means that in all these instances the conversion problem is completely solved, giving rise to the rate

$$R(\psi^{ABC} \to \phi^{ABC}) = \min\left\{\frac{S(\psi^A)}{S(\phi^A)}, \frac{S(\psi^B)}{S(\phi^B)}, \frac{S(\psi^C)}{S(\phi^C)}\right\}. \tag{13}$$

Moreover, the bound in Eq. (7) can be immediately generalized by interchanging the roles of the parties, i.e.,

$$R(\psi^{ABC} \to \phi^{ABC}) \geq \min\left\{\frac{S(\psi^B)}{S(\phi^A) + S(\phi^C)}, \frac{S(\psi^A)}{S(\phi^A)}, \frac{S(\psi^C)}{S(\phi^C)}\right\}, \tag{14}$$

$$R(\psi^{ABC} \to \phi^{ABC}) \geq \min\left\{\frac{S(\psi^C)}{S(\phi^A) + S(\phi^B)}, \frac{S(\psi^A)}{S(\phi^A)}, \frac{S(\psi^B)}{S(\phi^B)}\right\}. \tag{15}$$

The best bound is obtained by taking the maximum of Eqs. (7), (14), and (15).

Our results also shed new light on reversibility questions for tripartite state transformations. In general, a transformation $\psi \to \phi$ is said to be reversible if the conversion rates fulfill the relation

$$R(\psi \to \phi) = R(\phi \to \psi)^{-1}. \tag{16}$$

Let now ψ and ϕ be two states for which the bound in Theorem 1 is tight, e.g., $R(\psi \to \phi) = S(\psi^A)/S(\phi^A)$. Due to Eq. (3) it must be that $S(\psi^A)/S(\phi^A) \leq S(\psi^B)/S(\phi^B)$ in this case. If this inequality is strict (which will be the generic case), we obtain for the inverse transformation $\phi \to \psi$

$$R(\phi \to \psi) \leq \frac{S(\phi^B)}{S(\psi^B)} < \frac{S(\phi^A)}{S(\psi^A)} = R(\psi \to \phi)^{-1}, \tag{17}$$

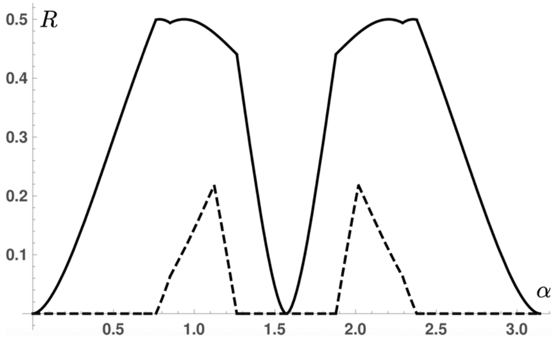

FIG. 2. Lower bound for the conversion rate from the state vector $|\psi\rangle^{ABC}$ in Eq. (18) into a GHZ state, obtained by taking the maximum of Eqs. (7), (14), and (15) [solid line] and the difference between upper bound (3) and lower bound [dashed line] for $\beta = 1/2$.

where the first inequality follows from Eq. (3). These results show that those states that saturate the bound (3) do not allow for reversible transformations in the generic case.

We will now comment on the limits of the approach presented here. In particular, it is important to note that the lower bound in Theorem 1 is not optimal in general. This can be seen in the most simple way by considering the trivial transformation which leaves the state unchanged, i.e., $\psi^{ABC} \to \psi^{ABC}$. Clearly, this can be achieved with unit rate $R = 1$. However, if we apply the lower bound in Theorem 1 to this transformation, we get $R \geq S(\psi^A)/[S(\psi^B) + S(\psi^C)]$. Due to subadditivity, it follows that our lower bound is in general below the achievable unit rate in this case. As an example, consider the family of state vectors

$$|\psi\rangle^{ABC} = \cos\alpha|0,0,0\rangle + \sin\alpha\sin\beta|0,1,1\rangle + \sin\alpha\cos\beta|1,0,1\rangle, \tag{18}$$

for real α, β which we aim to convert into the GHZ state. The solid line in Fig. 2 shows our lower bound, taking the maximum of Eqs. (7), (14), and (15) as a function of α for $\beta = 1/2$. The dashed line in Fig. 2 depicts the difference between the upper bound (3) and our lower bound. Note that the bounds coincide for a large parameter range of α, implying that our bound gives the exact conversion rate in these cases. To the best of our knowledge, this outperforms any previously known bounds, such as the longstanding one of Smolin *et al.* [22] as they consider only one-way broadcasting protocols while ours is not limited to a particular class of LOCC.

Multipartite pure states.—In the discussion so far, we have focused on tripartite pure states. However, the presented tools can readily be applied to more general scenarios involving an arbitrary number of parties. In this more general setup the parties will be called Alice (A) and N Bobs (B_i) with $1 \leq i \leq N$. The aim of the process in this case is the asymptotic conversion of the $(N+1)$-partite pure state $\psi = \psi^{AB_1,\ldots,B_N}$ into the state $\phi = \phi^{AB_1,\ldots,B_N}$. The general idea for this procedure follows the same line of reasoning as in the tripartite scenario discussed above. In the first step, entanglement combing is applied to the state ψ, i.e., the transformation

$$\psi \to \mu_1^{A_1B_1} \otimes \mu_2^{A_2B_2} \otimes \cdots \otimes \mu_N^{A_NB_N} \tag{19}$$

with pure states μ_i. In the next step, Alice and the first Bob B_1 transform their state $\mu_1^{A_1B_1}$ into the desired final state ϕ via bipartite LOCC. In the final step, Alice applies Schumacher compression to parts of her state ϕ, and sends these parts to each of the remaining Bobs $B_2, \ldots, B_N$ by using entanglement obtained in the first step of this protocol. As in the tripartite case, this protocol can be further optimized by interchanging the roles of the parties and applying timesharing.

Theorem 2: lower bound for multipartite state conversion.—For $N+1$-partite pure states $\psi^{AB_1,\ldots,B_N}$ and $\phi^{AB_1,\ldots,B_N}$, the LOCC conversion rate is bounded from below as

$$R(\psi^{AB_1,\ldots,B_N} \to \phi^{AB_1,\ldots,B_N}) \geq \min_X\left\{\frac{S(\psi^{AX})}{\sum_{B_i\notin X} S(\phi^{B_i})}\right\}, \tag{20}$$

where X denotes a subsystem of all Bobs, including the empty set.

The theorem is proven in the Supplemental Material [18]. By using similar arguments as below Eq. (3), an upper bound to the conversion rate is found to be

$$R(\psi^{AB_1,\ldots,B_N} \to \phi^{AB_1,\ldots,B_N}) \leq \min_i \frac{S(\psi^{B_i})}{S(\phi^{B_i})}. \tag{21}$$

The bounds in Eqs. (20) and (21) coincide if the following equality holds true for some $1 \leq i \leq N$,

$$\frac{S(\psi^{B_i})}{S(\phi^{B_i})} = \min_X\left\{\frac{S(\psi^{AX})}{\sum_{B_j\notin X} S(\phi^{B_j})}\right\}. \tag{22}$$

In those instances, Theorem 2 leads to a full solution of the conversion problem, and the corresponding rate is given by

$$R(\psi^{AB_1,\ldots,B_N} \to \phi^{AB_1,\ldots,B_N}) = \min_i \frac{S(\psi^{B_i})}{S(\phi^{B_i})}. \tag{23}$$

As in the tripartite case, the bound of Eq. (20) can be generalized by interchanging the roles of Alice and different Bobs.

Generalization to multipartite mixed states.—We will now show that the ideas which led to lower bounds on conversion rates in the previous sections can also be used in this mixed-state scenario. We will demonstrate this on a specific example, considering the transformation

$$|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}| \to \sigma, \tag{24}$$

where $|\mathrm{GHZ}\rangle = (|0\rangle^{\otimes N+1} + |1\rangle^{\otimes N+1})/\sqrt{2}$ denotes an $(N+1)$-partite GHZ state vector, and $\sigma = \sigma^{AB_1,\ldots,B_N}$ is an arbitrary $(N+1)$-partite mixed state. As we show in the Supplemental Material [18], by using similar methods as in previous sections, we obtain a lower bound on the transformation rate,

$$R(|\mathrm{GHZ}\rangle\langle\mathrm{GHZ}| \to \sigma) \geq \frac{1}{E_{\mathrm{c}}^{A|B_1,\ldots,B_N}(\sigma) + \sum_{j=3}^{N} S(\sigma^{B_j})}, \tag{25}$$

where $E_{\mathrm{c}}^{A|B_1,\ldots,B_N}$ denotes the entanglement cost [23] between Alice and all the other Bobs.

The upper bound (21) for the transformation rate R can be generalized as [see Eq. (146) in Ref. [1]]

$$R(\rho \to \sigma) \leq \min_{\mathcal{P}} \frac{E_{\infty}^{\mathcal{P}|\bar{\mathcal{P}}}(\rho)}{E_{\infty}^{\mathcal{P}|\bar{\mathcal{P}}}(\sigma)}. \tag{26}$$

Here, $E_\infty(\rho) = \lim_{n\to\infty} E_{\mathrm{r}}(\rho^{\otimes n})/n$ is the regularized relative entropy of entanglement [24,25], and $\mathcal{P}|\bar{\mathcal{P}}$ denotes a bipartition of all the $N+1$ subsystems [26].

Applications in quantum networks.—It should be clear that the results established here readily allow us to assess how resources for multipartite protocols can be prepared from multipartite states given in some form. Specifically, our techniques lead to optimal GHZ distillation rates for various classes of pure states and the lower bound can be computed easily for states of low dimension, rendering them useful for any quantum information processing tasks relying on GHZ states. In particular, GHZ states readily provide a resource for quantum secret sharing [8,9] in which a message is split into parts so that no subset of parties is able to access the message, while at the same time the entire set of parties is. It also gives rise to an efficient scheme of quantum secret sharing requiring purely classical communication during the reconstruction phase [27].

One motivation of our endeavor stems from the need to establish how multipartite resources for protocols beyond point-to-point schemes in quantum networks can be prepared and manipulated. Indeed, point-to-points schemes focus on the formation of Bell states through a network to perform bipartite applications such as quantum key distribution. However, distributing multipartite resources is mandatory for quantum protocols such as quantum secret sharing [8,9], quantum voting [28], or distributed quantum computing [29] which exploit the natural multipartite entanglement of a quantum network. We expect the study of multipartite resources to be particularly important when thinking of applications such as transforming resources into the desired form in quantum networks [7]. Here, multipartite entanglement is conceived to be created by local processes and bipartite transmissions involving pairs of nodes, followed by steps of entanglement manipulation, which presumably involve instances of classical routing techniques. Creating entanglement is undeniably costly in quantum networks. It has already been established on the single-shot level that nodes of a quantum network allow for quicker communication with fewer requirements concerning the channel capacity and memory than sharing bipartite pairs between nodes [30–32], specifically in networks featuring bottlenecks such as in the butterfly network. Building on these findings, it seems imperative that previously distributed states are reused as much as possible by converting them into desired resources rather than using more precious entanglement, in schemes involving more than one copy at a time. The bounds above can, e.g., be readily applied to the setting in which the preparation of smaller graph states has been successful [30–32], but from which larger GHZ states are still to be built up without wanting to discard previously prepared steps. We hope that our established bounds provide meaningful guidance as to how to manage and recycle resources for quantum networks.

Conclusions.—In this Letter, we have reported substantial progress on asymptotic state transformation via multipartite local operations and classical communication, tackling an important long-standing problem that to a large extent remained open since the early development of quantitative entanglement theory [4]. Similar techniques may also prove helpful in the study of other quantum resource theories different from entanglement, such as the resource theory of quantum coherence [33] and quantum thermodynamics [34,35]. Putting notions of entanglement combing into a fresh light, we have been able to derive stringent bounds on multipartite entanglement transformations. This progress seems particularly relevant in the light of the advent of quantum networks and the quantum internet in which multipartite features are directly exploited beyond point-to-point architectures. It is the hope that the present work stimulates further progress in the understanding of multipartite protocols.

We acknowledge discussions with Paweł Horodecki and financial support by the Alexander von Humboldt-Foundation, the National Science Center in Poland (POLONEZ UMO-2016/21/P/ST2/04054), the BMBF (Q.com, Q.Link.X), and the ERC (TAQ). This work was further supported by the "Quantum Coherence and Entanglement for Quantum Technology" project, carried out within the First Team programme of the Foundation for Polish Science cofinanced by the European Union under the European Regional Development Fund.

[1] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[2] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A **53**, 2046 (1996).

[3] M. Walter, D. Gross, and J. Eisert, arXiv:1612.02437.
[4] C. H. Bennett, S. Popescu, D. Rohrlich, J. A. Smolin, and A. V. Thapliyal, Phys. Rev. A **63**, 012307 (2000).
[5] J. I. de Vicente, C. Spee, D. Sauerwein, and B. Kraus, Phys. Rev. A **95**, 012323 (2017).
[6] S. Ishizaka and M. B. Plenio, Phys. Rev. A **72**, 042325 (2005).
[7] H. J. Kimble, Nature (London) **453**, 1023 (2008).
[8] M. Hillery, V. Bužek, and A. Berthiaume, Phys. Rev. A **59**, 1829 (1999).
[9] R. Cleve, D. Gottesman, and H.-K. Lo, Phys. Rev. Lett. **83**, 648 (1999).
[10] P. Vrana and M. Christandl, J. Math. Phys. (N.Y.) **56**, 022204 (2015).
[11] P. Vrana and M. Christandl, Commun. Math. Phys. **352**, 621 (2017).
[12] D. Yang and J. Eisert, Phys. Rev. Lett. **103**, 220501 (2009).
[13] M. Horodecki, J. Oppenheim, and A. Winter, Nature (London) **436**, 673 (2005).
[14] M. Horodecki, J. Oppenheim, and A. Winter, Commun. Math. Phys. **269**, 107 (2007).
[15] D. DiVincenzo, C. Fuchs, H. Mabuchi, J. Smolin, A. Thapliyal, and A. Uhlmann, in *Quantum Computing and Quantum Communications*, Lecture Notes in Computer Science Vol. 1509 (Springer, Berlin, Heidelberg, 1999), pp. 247–257.
[16] J. A. Smolin, F. Verstraete, and A. Winter, Phys. Rev. A **72**, 052317 (2005).
[17] N. Linden, S. Popescu, B. Schumacher, and M. Westmoreland, Quantum Inf. Process. **4**, 241 (2005).
[18] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.125.080502 for details of the proofs of the main text.
[19] B. Schumacher, Phys. Rev. A **51**, 2738 (1995).
[20] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[21] S. Pirandola, J. Eisert, C. Weedbrook, A. Furusawa, and S. L. Braunstein, Nat. Photonics **9**, 641 (2015).
[22] J. A. Smolin, F. Verstraete, and A. Winter, Phys. Rev. A **72**, 052317 (2005).
[23] P. M. Hayden, M. Horodecki, and B. M. Terhal, J. Phys. A **34**, 6891 (2001).
[24] K. Audenaert, J. Eisert, E. Jane, M. B. Plenio, S. Virmani, and B. De Moor, Phys. Rev. Lett. **87**, 217902 (2001).
[25] A. Winter, Commun. Math. Phys. **347**, 291 (2016).
[26] If there is a bipartition $\mathcal{P}|\bar{\mathcal{P}}$ with $E_\infty^{\mathcal{P}|\bar{\mathcal{P}}}(\rho) = E_\infty^{\mathcal{P}|\bar{\mathcal{P}}}(\sigma) = 0$, this bipartition is not taken into account in Eq. (26).
[27] A. Broadbent, P. R. Chouha, and A. Tapp, in *2009 Third International Conference on Quantum, Nano and Micro Technologies* (IEEE, 2009), https://ieeexplore.ieee.org/document/4782923.
[28] N. Bao and N. Yunger Halpern, Phys. Rev. A **95**, 062306 (2017).
[29] H. J. Briegel, D. E. Browne, W. Dür, R. Raussendorf, and M. Van den Nest, Nat. Phys. **5**, 19 (2009).
[30] E. Schoute, L. Mancinska, T. Islam, I. Kerenidis, and S. Wehner, arXiv:1610.05238.
[31] A. Dahlberg and S. Wehner, Phil. Trans. R. Soc. A **376**, 20170325 (2018).
[32] F. Hahn, A. Pappa, and J. Eisert, npj Quantum Inf. **5**, 76 (2019).
[33] A. Streltsov, G. Adesso, and M. B. Plenio, Rev. Mod. Phys. **89**, 041003 (2017).
[34] M. Lostaglio, D. Jennings, and T. Rudolph, Nat. Commun. **6**, 6383 (2015).
[35] P. Ćwikliński, M. Studziński, M. Horodecki, and J. Oppenheim, Phys. Rev. Lett. **115**, 210403 (2015).